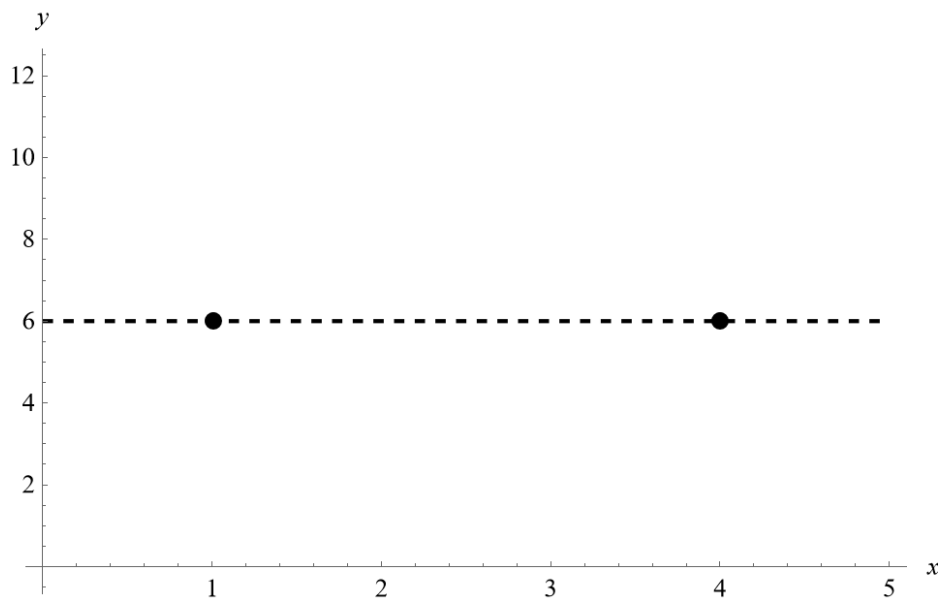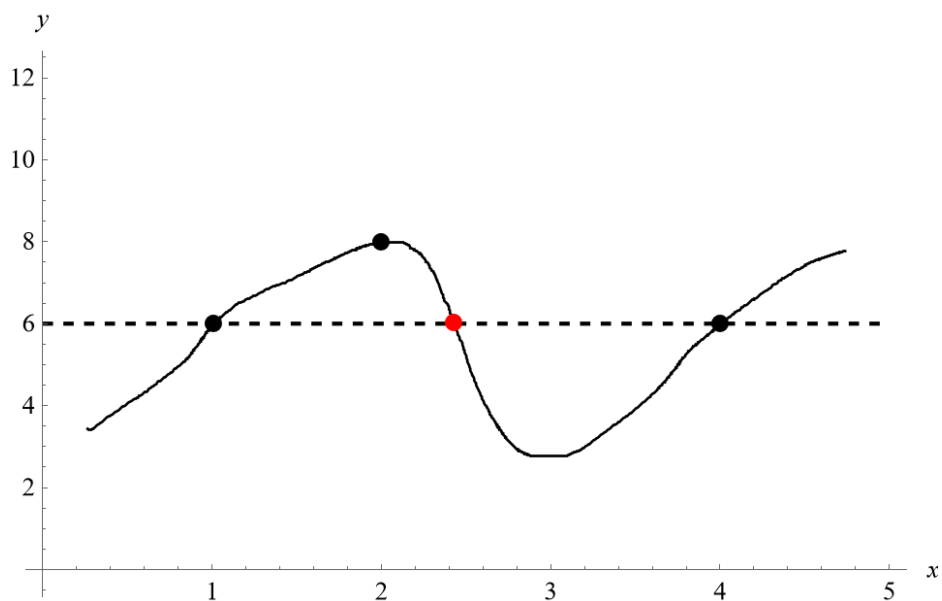## Exercise 52

Suppose f is continuous on $[1, 5]$ and the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.

Solution

The fact that there are two solutions to $f(x) = 6$ means that the function passes through $y = 6$ twice—once at $x = 1$ and once at $x = 4$.

If the function is continuous on $[1, 5]$ and the function is 8 when $x = 2$, then look what happens if the function is less than 6 at $x = 3$.

If $f(3) < 6$, then there would be a third solution to the equation $f(x) = 6$, indicated by the red dot. This would contradict the assumption that only $x = 1$ and $x = 4$ are solutions to $f(x) = 6$. It's necessary, then, that $f(3) > 6$ if $f(2) = 8$.